Math 150: Final Exam **Name:** _____

Instructions: You must show your work on all problems except problem 1. If you write only the answer, you will get zero credit for that problem.

1. (7 points) Decide whether the following are true or false. Write out the whole word "true" or "false"; do not write just "T" or "F".

(a) _____ The function $f(x) = \begin{cases} 0, & x < 0 \\ \sin x, & x \geq 0 \end{cases}$ is continuous.

(b) _____ If a function is continuous, then it is differentiable.

(c) _____ If $f'(x) = g'(x)$, then $f(x) = g(x)$.

(d) _____ If $f'(a) = 0$, then f has a local minimum or a local maximum at $x = a$.

(e) _____ If f is continuous on (a, b), then there is a point x_0 such that $f(x_0)$ is an absolute maximum of f on (a, b).

(f) _____ If $f'(a) = 0$ and $f''(a) > 0$, then f has a global minimum at a.

(g) _____ If $f(x) < 0$ on $[a, b]$, then $\int_a^b f(x)\,dx < 0$.

2. (5 points) Evaluate $\lim_{x \to \infty} \dfrac{1 - x^2}{x^3 - x + 1}$.

3. (5 points) Find an equation of the tangent line to the curve $y = \sin(\sin x)$ at the point $(\pi, 0)$.

4. (10 points) If a sphere grows so that its surface area increases at a rate of 2 cm^2/min, find the rate at which the radius increases when the radius is 20 cm. (Note that the volume of a sphere is $\frac{4}{3}\pi r^3$ and its surface area is $4\pi r^2$.)

5. (10 points) Let $f(x) = x^3 - 2x^4$. Find the intervals where f increases and decreases and where f is concave up and concave down. Also find the local minimums and local maximums and the inflection points.

6. (5 points) Evaluate $\displaystyle\int \frac{1}{\cos^2 t\sqrt{1+\tan t}}\,dt$. (Hint: Think about trig identities for $\sec t$.)

7. (5 points) Evaluate $\displaystyle\int_0^{\pi/8} \sec 2\theta \tan 2\theta\,d\theta$.

8. (10 points) Find the area of the region enclosed by $y = |x|$ and $y = x^2$. (Hint: Sketch a graph and consider any symmetry.)

9. (10 points) Let R be the region enclosed by the curves $y = x^3$ and $y = \sqrt{x}$. What is the volume of the solid of revolution formed by rotating R about the x-axis?