Review Problems for the Final Exam

(Note that all problems are odd-numbered problems from the textbook, so the answers are in the back of the book.)

Theory

Decide whether the following statements are true or false:

1. If a function is differentiable, then it is continuous.
2. If a function is continuous, then it is differentiable.
3. If f is differentiable, then f' is continuous.
4. If the graph of f has a sharp corner at $x = a$, then f is not differentiable at a.
5. If $f'(x) = g'(x)$, then $f(x) = g(x)$.
6. If $f'(a) = 0$, then there is a local minimum or local maximum at $x = a$.
7. If f is an even function, then $\int_{-a}^{a} f(x)\,dx = 0$.
8. For any function f, $\int_a^b f(x)\,dx = \int_a^b f(p)\,dp$.
9. If $f'(x) > 0$ for all x, then f is increasing everywhere.
10. If F is an antiderivative of f, and $f(x) > 0$ for all x, then F is increasing everywhere.

Answers: true, false, false, true, false, false, false, true, true, true.

Limits

Evaluate the following:

1.4.13 $\displaystyle\lim_{x\to 5} \frac{x^2 - 5x + 6}{x - 5}$

1.4.21 $\displaystyle\lim_{h\to 0} \frac{\sqrt{9+h} - 3}{x - 5}$

1.4.49 $\displaystyle\lim_{x\to 0} \frac{\sin 3x}{x}$

1.4.51 $\displaystyle\lim_{x\to 0} \frac{\tan 6t}{\sin 2t}$

1.6.21 $\displaystyle\lim_{t\to\infty} \frac{\sqrt{t} + t^2}{2t - t^2}$

1.6.27 $\lim_{x\to\infty} \dfrac{x^4 - 3x^2 + x}{x^3 - x + 2}$

Derivatives

Differentiate the following:

2.4.21 $f(\theta) = \dfrac{\sec\theta}{1 + \sec\theta}$

2.4.23 $y = \dfrac{t\sin t}{1 + t}$

2.5.7 $F(x) = (x^4 + 3x^2 - 2)^5$

2.5.25 $y = \dfrac{r}{\sqrt{r^2 + 1}}$

2.5.27 $y = \sin\sqrt{1 + x^2}$

2.5.43 $y = \cos(x^2)$

Chapter 2 review, 17 $y = x^2 \sin \pi x$

Chapter 2 review, 21 $y = \tan\sqrt{1 - x}$

Chapter 2 review, 35 $y = \tan^2(\sin\theta)$

2.4.35 If $H(\theta) = \theta\sin\theta$, find $H'(\theta)$ and $H''(\theta)$.

Implicit differentiation

2.6.5 Find $\frac{dy}{dx}$ where $x^2 + xy - y^2 = 4$.

2.6.7 Find $\frac{dy}{dx}$ where $y\cos x = x^2 + y^2$.

2.6.15 Find $\frac{dy}{dx}$ where $y\cos x = 1 + \sin(xy)$.

Related rates

2.7.7 Each side of a square is increasing at a rate of 6 cm/s. At what rate is the area of the square increasing when the area of the square is 16 cm^2?

2.7.25 A trough is 10 ft long and its ends have the shape of isosceles triangles that are 3 ft across at the top and have a height of 1 ft. If the trough is being filled with water at a rate of 12 ft^3/min, how fast is the water level rising when the water is 6 inches deep?

2.7.29 Two sides of a triangle are 4 m and 5 m in length and the angle between them is increasing at a rate of 0.06 radian/s. Find the rate at which the area of the triangle is increasing when the angle between the sides of fixed length is $\frac{\pi}{3}$.

Chapter 2 review, 69 A balloon is rising at a constant speed of 5 ft/s. A boy is cycling along a sraight road at a speed of 15 ft/s. When he passes under the balloon, it is 45 ft above him. How fast is the distance between the boy and the balloon increasing 3 s later?

Optimization and curve sketching

3.1.35 Find the absolute maximum and the absolute minimum of $f(x) = 12 + 4x - x^2$ on $[0, 5]$.

3.1.47 Find the absolute maximum and the absolute minimum of $f(x) = x^5 - x^3 + 2$ on $[-1, 1]$.

3.3.21 For $f(x) = x^3 - 12x + 2$, find the intervals where f is increasing/decreasing, the local maximum and minimum values, the intervals where f is concave up/down, and the inflection points.

3.3.25 For $h(x) = (x + 1)^5 - 5x - 2$, find the intervals where h is increasing/decreasing, the local maximum and minimum values, the intervals where h is concave up/down, and the inflection points.

3.3.29 For $C(x) = x^{\frac{1}{3}}(x + 4)$, find the intervals where C is increasing/decreasing, the local maximum and minimum values, the intervals where C is concave up/down, and the inflection points.

3.4.9 Sketch the curve of $y = \frac{x-1}{x^2}$ by first finding asymptotes, intervals where it's increasing/decreasing, and concavity.

3.4.11 Sketch the curve of $y = \frac{1}{x^2-9}$ by first finding asymptotes, intervals where it's increasing/decreasing, and concavity.

3.5.11 If 1200 cm^2 of material is available to make a box with a square base and open top (but not necessarily square sides), find the largest possible volume of the box.

3.5.25 A Norman window has the shape of a rectangle surmounted by a semicircle. (Thus the diameter of the semicircle is equal to the width of the rectangle.) If the perimeter of the window is 30 ft, find the dimensions of the window so that the greatest possible amount of light is admitted (i.e., greatest area).

3.5.27 A piece of wire 10 m long is cut into two pieces. One piece is bent into a square and the other is bent into an equilateral triangle. How show the wire be cut so that the total area enclosed is (a) maximized? (b) minimized?

Integrals

Evaluate the following:

4.3.9 $\displaystyle\int_1^4 \left(\frac{4+6u}{\sqrt{u}}\right) du$

4.3.13 $\displaystyle\int_1^4 \sqrt{\frac{5}{x}}\, dx$

4.3.47 $\displaystyle\int \frac{\sin x}{1-\sin^2 x}\, dx$

4.5.17 $\displaystyle\int \sec^2\theta \tan^3\theta\, d\theta$

4.5.23 $\displaystyle\int \sqrt{\cot x}\csc^2 x\, dx$

4.5.27 $\displaystyle\int \frac{z^2}{\sqrt[3]{1+z^3}}\, dz$

4.5.33 $\displaystyle\int_0^\pi \sqrt[3]{1+7x}\, dx$

Chapter 4 review, 19 $\displaystyle\int_{-\pi/4}^{\pi/4} \frac{t^4\tan t}{2+\cos t}\, dt.$

4.4.9 Find the derivative of $h(x) = \displaystyle\int_2^{1/x} \sin^4 t\, dt.$

4.4.13 Find the derivative of $g(x) = \displaystyle\int_{2x}^{3x} \frac{u^2-1}{u^2+1}\, du.$

Areas and volumes

7.1.1 Find the area of the region enclosed by the curves $y = x$ and $y = 5x - x^2$.

7.2.5 Let R be the region enclosed by the curves $y = x^3$, $y = x$, and $x \geq 0$. Find the volume of the solid obtained by rotating R about the x-axis.

7.2.9 Let R be the region enclosed by the curves $y = x$ and $y = \sqrt{x}$. Find the volume of the solid obtained by rotating R about the line $y = 1$.