Review Problems for Quiz 1

(Note that all problems are odd-numbered problems from the textbook, so the answers are in the back of the book.)

1 Limits and continuity

Evaluate the following limits or show that they do not exist:

1.4.15 $\lim_{t\to -3} \dfrac{t^2-9}{2t^2+7t+3}$

1.4.17 $\lim_{h\to 0} \dfrac{(-5+h)^2-25}{h}$

1.4.21 $\lim_{h\to 0} \dfrac{\sqrt{9+h}-3}{h}$

1.4.49 $\lim_{x\to 0} \dfrac{\sin 3x}{x}$

1.4.33 If $4x-9 \le f(x) \le x^2-4x+7$ for $x \ge 0$, find $\lim_{x\to 4} f(x)$.

1.5.31 Find the numbers at which the following function is discontinuous:

$$f(x) = \begin{cases} x+2 & \text{if } x < 0 \\ 2x^2 & \text{if } 0 \le x \le 1 \\ 2-x & \text{if } x > 1 \end{cases}.$$

1.5.29 Show that the following function is continuous:

$$f(x) = \begin{cases} x^2 & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \ge 1 \end{cases}.$$

1.6.3 Sketch a graph of a function satisfying all the following criteria:

$$\lim_{x\to 0} f(x) = -\infty \qquad \lim_{x\to -\infty} f(x) = 5 \qquad \lim_{x\to \infty} f(x) = -5.$$

Find the following limits or say why they don't exist:

1.6.19 $\lim_{x\to \infty} \dfrac{3x-2}{2x+1}$

1.6.23 $\lim_{x\to \infty} \dfrac{(2x^2+1)^2}{(x-1)^2(x^2+x)}$

1.6.27 $\lim_{x\to\infty} \cos x$

2 Derivatives

Compute the derivatives of the following without using the definition of the derivative:

2.3.19 $y = \dfrac{x^2 + 4x + 3}{\sqrt{x}}$

2.4.7 $f(x) = \sin x + \frac{1}{2}\cot x$

2.4.13 $y = \dfrac{x^3}{1 - x^2}$

2.4.17 $f(t) = \dfrac{2t}{2 + \sqrt{t}}$

2.5.5 $y = \sqrt{\sin x}$

2.5.9 $F(x) = \sqrt{1 - 2x}$

2.5.19 $h(t) = (t + 1)^{\frac{2}{3}}(2t^2 - 1)^3$

2.5.29 $f(x) = \sin(\tan 2x)$

2.5.33 $y = \left(\dfrac{1 - \cos 2x}{1 + \cos 2x}\right)^4$

2.5.43 Find the second derivative of $y = \cos(x^2)$.

2.5.45 Find the second derivative of $H(t) = \tan 3t$.

2.3.27 Find the equation of the tangent line to $y = 6\cos x$ at the point $(\frac{\pi}{3}, 3)$.

2.4.27 Find the equation of the tangent line to $y = \frac{x^2-1}{x^2+x+1}$ at the point $(1, 0)$.

2.3.39 Show that the curve $y = 6x^3 + 5x - 3$ has no tangent line with a slope of 4.

2.5.55 Here are some values of f, g, f', and g' at certain values of x:

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
1	3	2	4	6
2	1	8	5	7
3	7	2	7	9

Find $(f \circ g)'(1)$ and $(g \circ f)'(1)$.