Math 150: Quiz 3 **Name:** ______________________________

Instructions: You must show your work on all problems. If you write only the answer, you will get zero credit for that problem.

1. (5 points) Find the most general antiderivative of $f(x) = \dfrac{x^4 + 3\sqrt{x}}{x^2}$. (Hint: Divide first, then compute the antiderivative.)

$$f(x) = x^2 + 3x^{-3/2}$$

$$F(x) = \frac{1}{3}x^3 + 3(-2)x^{-1/2} + C$$

$$= \frac{1}{3}x^3 - \frac{6}{\sqrt{x}} + C$$

2. (5 points) If $f''(\theta) = \sin\theta + \cos\theta$, $f(0) = 3$, and $f'(0) = 4$, find $f(\theta)$.

$$f'(\theta) = -\cos\theta + \sin\theta + C_1$$

$$4 = f'(0) = -1 + 0 + C_1 \Rightarrow C_1 = 5$$

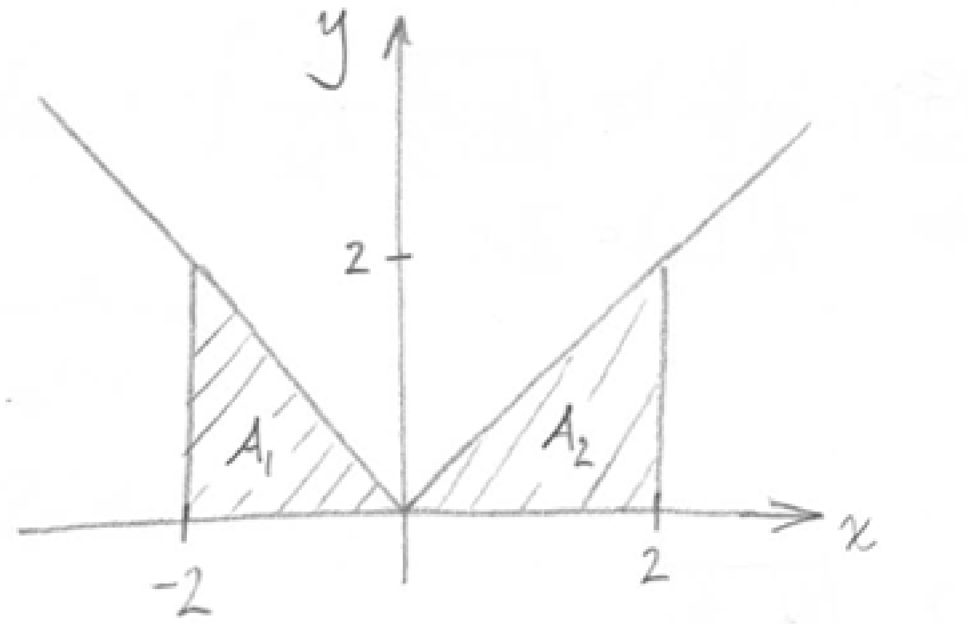
$$f'(\theta) = -\cos\theta + \sin\theta + 5$$

$$f(\theta) = -\sin\theta - \cos\theta + 5\theta + C_2$$

$$3 = f(0) = -0 - 1 + 5\cdot 0 + C_2 \Rightarrow C_2 = 4$$

$$f(\theta) = -\sin\theta - \cos\theta + 5\theta + 4$$

3. (5 points) Evaluate $\int_{-2}^{2} |x|\, dx$ by interpreting the integral as the area under the curve and then using basic geometry. For this problem only, you are not allowed to use the Evaluation Theorem (aka Net Change Theorem aka Fundamental Theorem of Calculus Part 2). (Hint: It might help to sketch a graph.)



$$\int_{-2}^{2} |x|\, dx = A_1 + A_2 = \frac{1}{2}\cdot 2\cdot 2 + \frac{1}{2}\cdot 2\cdot 2 = 4$$

4. (5 points) Evaluate $\int_{1}^{2} \left(\frac{1}{x^2} - \frac{4}{x^3}\right) dx$.

$$\int_{1}^{2} (x^{-2} - 4x^{-3})\,dx = \left[-x^{-1} - 4\left(-\tfrac{1}{2}\right)x^{-2}\right]_1^2$$

$$= \left[-\frac{1}{x} + \frac{2}{x^2}\right]_1^2$$

$$= \left(-\frac{1}{2} + \frac{2}{4}\right) - (-1+2)$$

$$= -1$$

5. (5 points) Find the derivative of $f(x) = \int_0^{x^2} \sqrt{1+r^3}\,dr$.

$g(x) = \int_0^x \sqrt{1+r^3}\,dr \Rightarrow g'(x) = \sqrt{1+x^3}$

$h(x) = x^2 \Rightarrow h'(x) = 2x$

$f(x) = g(h(x))$

$f'(x) = g'(h(x))\,h'(x) = 2x\sqrt{1+x^6}$

6. (5 points) Evaluate $\int \frac{x}{(x^2+1)^2}\,dx$.

$$\underbrace{\int \frac{x}{(x^2+1)^2}\,dx}_{\substack{\text{Let } u = x^2+1 \\ du = 2x\,dx}} = \int \frac{1}{u^2}\,\frac{1}{2}\,du = \frac{1}{2}(-1)u^{-1} + C = -\frac{1}{2(x^2+1)} + C$$

7. (5 points) Evaluate $\displaystyle\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx$.

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}\,dx = \int \sin u \cdot 2\,du = -2\cos u + C = -2\cos\sqrt{x} + C$$

Let $u = \sqrt{x}$

$du = \frac{1}{2\sqrt{x}}\,dx$

8. (5 points) Evaluate $\displaystyle\int_0^1 (2t-1)^{10}\,dt$.

$$\underbrace{\int_0^1 (2t-1)^{10}\,dt = \int_{-1}^{1} \frac{1}{2} u^{10}\,du}_{} = \frac{1}{2}\frac{1}{11} u^{11}\Big|_{-1}^{1} = \frac{1}{22}\left(1^{11} - (-1)^{11}\right) = \frac{2}{22} = \frac{1}{11}$$

Let $u = 2t-1$

$du = 2\,dt$

$u(0) = -1$

$u(1) = 1$

9. (5 points) Evaluate $\int_0^{\frac{\pi}{2}} \cos x \sin(\sin x)\, dx$.

$$\int_0^{\frac{\pi}{2}} \underbrace{\cos x \sin(\sin x)\, dx}_{\substack{\text{Let } u = \sin x \\ du = \cos x\, dx \\ u(0) = \sin 0 = 0 \\ u\left(\frac{\pi}{2}\right) = \sin\frac{\pi}{2} = 1}} = \int_0^1 \sin u\, du = -\cos u \Big|_0^1 = -\cos 1 + \cos 0$$

$$= -\cos 1 + 1$$

10. (5 points) Evaluate $\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x^4 \sin x\, dx$. (Hint: Is the integrand an even function or an odd function or neither?)

Since $(-x)^4 \sin(-x) = x^4(-\sin x) = -x^4 \sin x$,

the integrand is an odd function, so

$$\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} x^4 \sin x\, dx = 0.$$