Math 156: Exam 2 (take-home exam)
Due May 1. No late submissions will be accepted.

Instructions:

- *Write your solutions neatly, or else points will be deducted.*
- *You can take as much time as you need for the exam, and you can refer to your notes, textbooks, and your workshops.*
- *You cannot work with other students, ask anyone for help, or search for the problems online.*

1. (10 points)

Suppose A, B are sets. Prove $A \cap B = A - (A - B)$, by first showing $A \cap B \subseteq A - (A - B)$ and then showing $A \cap B \supseteq A - (A - B)$. (Venn diagrams do not suffice for a proof.)

2. (10 points)

Let $A = \{1, 2, 3, 4\}$. Give an example of

- a relation R_1 on A that is reflexive and symmetric but not transitive,
- a relation R_2 on A that is reflexive and transitive but not symmetric, and
- a relation R_3 on A that is symmetric and transitive but not reflexive.

3. (10 points)

Let $W = \{0, 1, 2, 3, \ldots\} = \{0\} \cup \mathbb{N}$. Define a relation R on $W \times W$ by $(x_1, x_2)\, R\, (y_1, y_2)$ if and only if $x_1 + y_2 = y_1 + x_2$. Prove R is an equivalence relation.

4. (10 points)

Let W and R be as defined in problem 3. Define C to be the set of equivalence classes of $W \times W$ under the equivalence relation R, i.e., $C = \Big\{ \big[(a, b)\big] : a, b \in W \Big\}$. Define the function $f : C \to \mathbb{Z}$ by $\big[(m, n)\big] \mapsto m - n$. Prove that f is injective and surjective.

5. (10 points)

Let W be as defined in problem 3. Define a function $f : W \times W \to \mathbb{Z}$ by $(a, b) \mapsto (-1)^a b$. Prove or disprove whether f is injective. Also prove or disprove whether f is surjective.

6. (10 points)

Suppose $f : A \to B$ and $Y_1, Y_2 \subseteq B$. Prove

$$f^{-1}(Y_1 \cap Y_2) = f^{-1}(Y_1) \cap f^{-1}(Y_2)$$

by first showing $f^{-1}(Y_1 \cap Y_2) \subseteq f^{-1}(Y_1) \cap f^{-1}(Y_2)$ and then showing $f^{-1}(Y_1 \cap Y_2) \supseteq f^{-1}(Y_1) \cap f^{-1}(Y_2)$.