Math 156: Workshop 4

Write your solutions neatly, or else points will be deducted. Prove the following.

1. (p.195 #2) For every $n \in \mathbb{N}$, we have

$$1^2 + 2^2 + 3^2 + \cdots + n^2 = \frac{n(n+1)(2n+1)}{6}.$$

2. (p.195 #6) For every $n \in \mathbb{N}$, we have

$$\sum_{i=1}^{n}(8i-5) = 4n^2 - n.$$

3. (p.195 #8) If $n \in \mathbb{N}$, then

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + \cdots + \frac{n}{(n+1)!} = 1 - \frac{1}{(n+1)!}.$$

4. (p.196 #16) For every $n \in \mathbb{N}$, we have $2^n + 1 \leq 3^n$.

5. (p.196 #26) Let F_n denote the nth term in the Fibonnaci sequence. Then

$$\sum_{k=1}^{n} F_k^2 = F_n F_{n+1}.$$

6. (p.196 #28) Let F_n denote the nth term in the Fibonnaci sequence. Then

$$F_2 + F_4 + F_6 + \cdots + F_{2n} = F_{2n+1} - 1.$$