Math 156: Workshop 7

Write your solutions neatly, or else points will be deducted. Prove the following.

1. (p.209 #12) Prove that the divisibility relation $a \mid b$ on the set $\mathbb{Z}$ is reflexive and transitive. Give a counterexample to show that it is not symmetric.

2. Define a relation on $\mathbb{R}$ as xRy if $|x - y| < 1$. Is R reflexive? Symmetric? Transitive? If a property holds, say why. If a property does not hold, give a counterexample.

3. (p.209 #16) Define a relation R on $\mathbb{Z}$ by declaring that xRy if and only if $x^2 \equiv y^2 \mod 4$. Prove that R is reflexive, symmetric, and transitive.

4. (p.214 #2) Let $A = \{a, b, c, d, e\}$. Suppose R is an equivalence relation on A. Suppose R has two equivalence classes. Also aRd, bRc, and eRd. Write out R as a set.

5. (p.214 #6) There are five different equivalence relations on the set $A = \{a, b, c\}$. Describe them all as partitions of A.

6. (p.217 #4) Suppose P is a partition of a set A. Define a relation R on A by declaring xRy if and only if $x, y \in X$ for some $X \in P$. Prove that R is an equivalence relation on A. Then prove that P is the set of all equivalence classes of R.