

Math 156: Workshop 9

Write your solutions neatly, or else points will be deducted.

1. Prove that a composition of two injective functions is injective, and that a composition of two surjective functions is surjective. Conclude a similar fact about bijections.
2. Explicitly describe a bijection showing that $\{0, 1\} \times \mathbb{N}$ and \mathbb{Z} have the same cardinality.
3. Explicitly describe bijections that show that the following three sets have the same cardinality: the set of integers, the set of even integers, and the set of odd integers.
4. For a fixed $n \in \mathbb{N}$, prove that the set of all subsets of \mathbb{N} of size n has the same cardinality as \mathbb{N} , i.e.,

$$\left| \{X \subseteq \mathbb{N} : |X| = n\} \right| = |\mathbb{N}|.$$

Then prove that the set of all finite subsets of \mathbb{N} has the same cardinality as \mathbb{N} , i.e.,

$$\left| \{X \subseteq \mathbb{N} : |X| \text{ is finite}\} \right| = |\mathbb{N}|.$$